Tales of Tails

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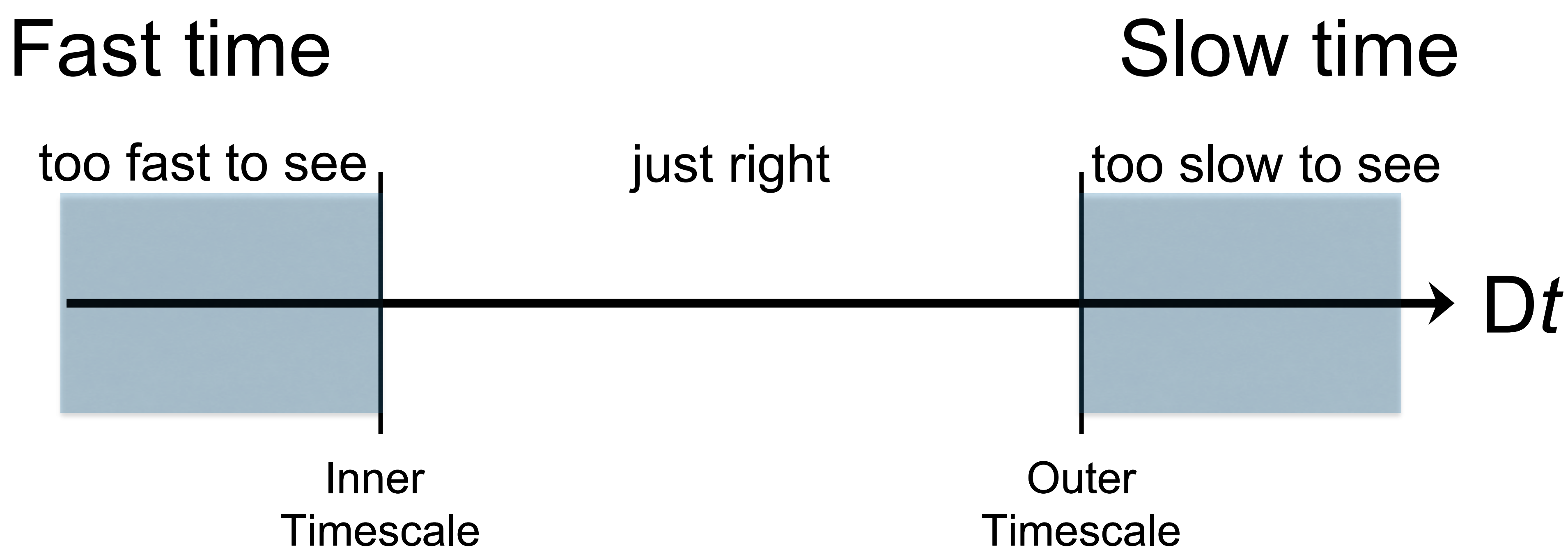
What is this?

Thermodynamics on very long timescales
Sun over parking lot (9 mo)

Niagara river

Timescales



<u>Too Fast</u>: D*t* too small to distinguish order of events, or too short an appearance to register at all.

<u>Too Slow</u>: D*t* too large to see, notice, or care about both events

Similarly in spatial dimensions: Coastline of Denmark = 8,750 km – with which resolution?

Distributions of distributions – wind

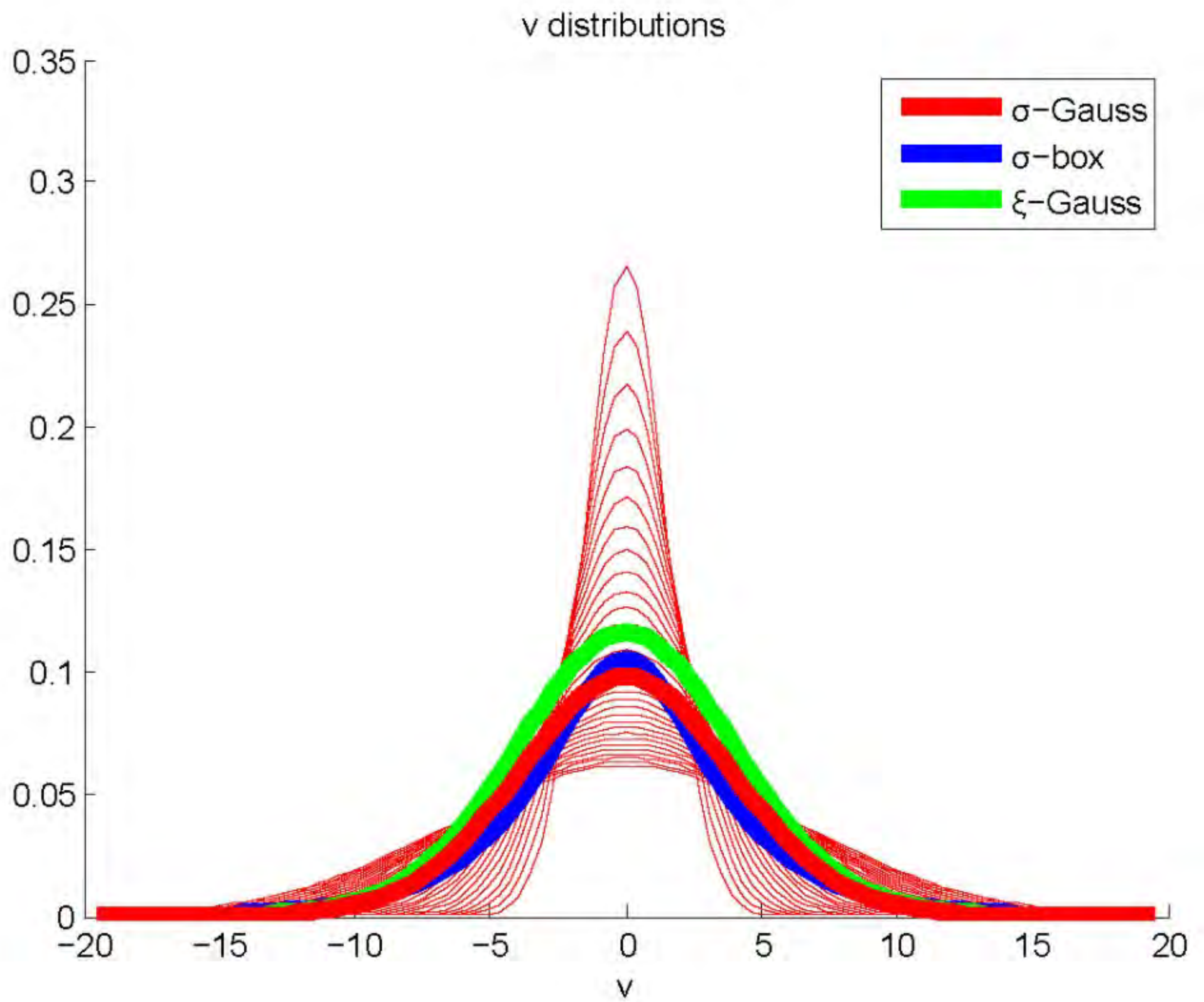
Mean air velocity (wind) fluctuates and turns into a slightly increased temperature, “thermalizing” wind:

$$p(v; u, T) = \left(\frac{m}{2kT}\right)^{1/2} \frac{1}{\sqrt{\pi}} e^{-\frac{m}{2kT}(v-u)^2}$$

$$p_u = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{u^2}{2\sigma_u^2}}$$

$$p(v; \theta) = \left(\frac{m}{2k\theta}\right)^{1/2} \frac{1}{\sqrt{\pi}} e^{-\frac{m}{2k\theta}v^2}$$

$$\theta = \frac{\sigma_u^2 m}{k} + T$$



Wind fluctuates too fast to see, thus no wind.

Distributions of distributions – temperature

By contrast, temperature fluctuations do **not** result in a new temperature, i.e. a new Maxwellian distribution:

$$p(v; w, \psi_0) = \int_{-\psi_0}^{\infty} p_{v\xi} p_\xi d\xi =$$

$$\left[\frac{1 + \mathrm{erf}\left(\frac{w^2 \psi_0}{(v^2+w^2)^{1/2}} \right)}{1 + \mathrm{erf}(w\psi_0)} \right] \frac{w^3 \psi_0}{\sqrt{\pi}(v^2 + w^2)^{3/2}} \, e^{-\frac{w^2 \psi_0^2 v^2}{v^2+w^2}} + \frac{1}{1 + \mathrm{erf}(w\psi_0)} \frac{w}{\pi(v^2 + w^2)} \, e^{-w^2 \psi_0^2}$$

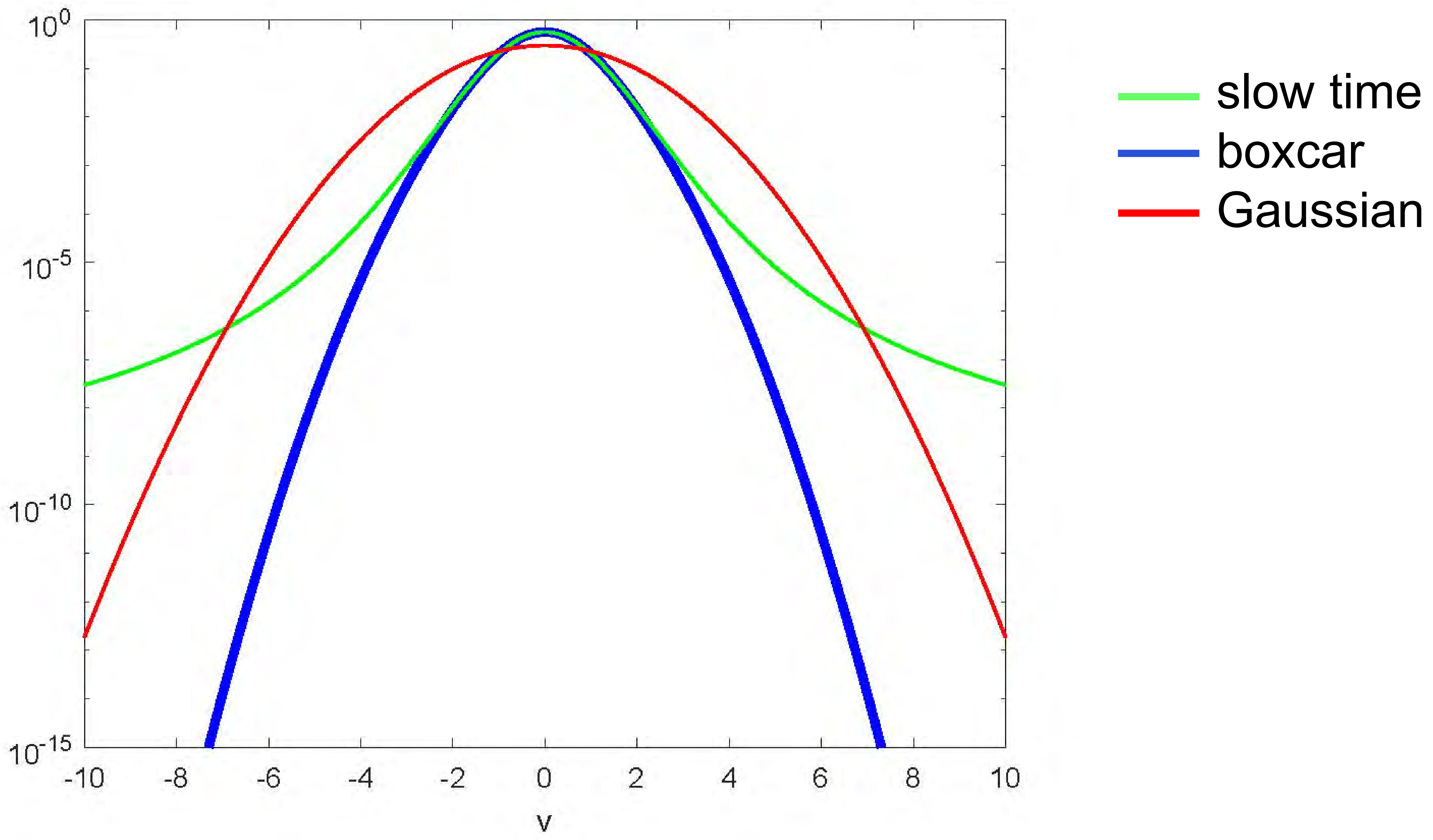
[*w* is the precision of the velocity precision]

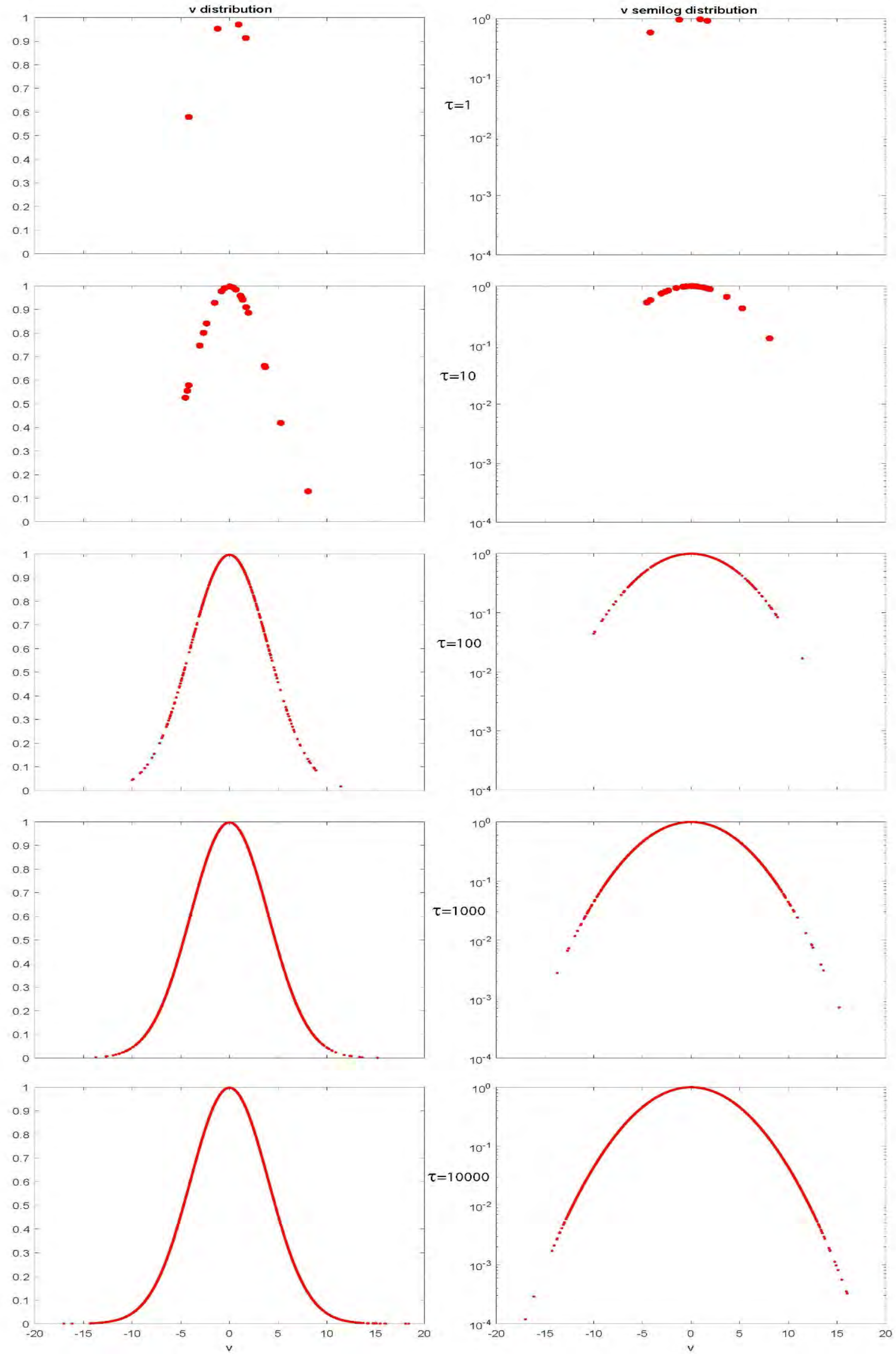
The result is polynomial tails of degree either -3 (infinite fluctuation domain) or -2 (semi-infinite domain).

This 'distribution' cannot be normalized.

No temperature – No state function – No equilibrium

Convoluted distributions compared to standard Maxwellian





Sampling a Gaussian distribution

All samplings are finite and never cover the entire tails.

Closure

- Do the new variables stand on their own, i.e. without reference to smaller (or larger) scales?
- Navier-Stokes differential equations do not on their own describe turbulent flow.
- $y_i = x_i^2$ does not imply $\langle y \rangle = \langle x \rangle^2$.
- Will thermodynamics extrapolate to slow time? On which observational timescale?
- Will the concept of equilibrium still be useful?
- LTE becomes less local at slow time.

Conclusions

- What is a state?
- The observational timescale has a profound influence on what is observed and which information it is possible (or of interest) to extract. The world looks very different depending on who is looking and what you are looking for.
- All thermodynamic functions may not exist at longer and shorter timescales.
- Local thermodynamic equilibrium (LTE) is based on variation of *intensive* variables, which may not persist.
- Climate is not merely an extrapolated weather forecast. The variables may not be the same.
- Closure is still an open question.