



# A Structure of Kinetic Hierarchy in Reaction Network



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### Institute for Chemical Reaction Design and Discovery

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### Institute for Chemical Reaction Design and Discovery https://www.icredd.hokudai.ac.jp/

#### Information Science AI, Machine Learning, Network theory, Information Theory, Applied Mathematics

### **Computational Science** Quantum Chemistry

MD simulation

**Experimental Science** Synthetic Chemistry, Biology, Medicine



- 1. Background and Motivation
- 2. Transition Disconnectivity Graph
- 3.Network Transition States
- 4.Lumping Theory to Preserve Hierarchical Timescales5.Outlooks

#### Lagrangian Coherent Structure

H. Teramoto et al ., **Chaos 23**, 043107 (2013) P. Nag et al. **J. Chem. Phys. 141**, 104907(2014)

#### **Higher rank saddles**

Y. Nagahata et al. Phys. Rev. E
87,62817 (2013); ibid. 88, 42923 (2013)

#### **Bifurcation of NHIM**

C.-B.Li, T. Komatsuzaki, *J. Chem. Phys.* 130,124116, (2009)



#### Origin of Stochasticity in Transitions

Komatsuzaki et al. **PNAS 98** 7666 (2002), C.-B.Li et al., **PRL 97**, 028302 (2006); H. Teramoto et al., **PRL 106**, 054101 (2011); **115**, 093003 (2015); **Nonlinearity 28**, 2677 (2015); **PRL**, 054101 (2015)

#### **TS in Complex Network**

**Y. Nagahata**, S. Maeda, H. Teramoto, T. Horiyama, T. Taketsugu, T. Komatsuzaki et al. *JPCB* 120(8), 1961-1971 (2015).

#### multidimensional Energy Landscape Based on single molecule time series

A. Baba et al., **PNAS 104**,19297 (2007); **PCCP**, **13**, 1395 (2011); K. Kamagata et al., **JACS 134**, 11525 (2012); J.N. Taylor et al., **Sci. Rep. 5**, 9174 (2015) J. Alfermann et al., **Nature Chem. Bio. 13**, 1009 (2017)



#### **Phase Space in Wells**

A. Shojiguchi et al. **Phys. Rev. E 76**, 056205 (2007); **ibid.75**, 035204(R) (2007); H. Teramoto et al. **J. Chem. Phys. 129**, 094302 (2008); **Phys. Rev. E 78**, 017202 (2008); **Theor. Chem. Acc. 133**, 1571, (2014)(invited); Y. Mizuno et al., **Physica D 428**, (2021) (invited)



#### No-return TS in Dissipative systems



S. Kawai, T. Komatsuzaki, Phys. Chem. Chem. Phys. (Perspective) 13, 21217 (2011); Phys. Chem. Chem. Phys. 12, 15382 (2010); ibid. 12, 7626 (2010); ibid. 12, 7636 (2010); J. Chem. Phys. 134, 114523, (2011); ibid. 131, 224505 (2009) ibid. 131, 224506 (2009); Nagahata et al., J. Chem. Phys. (Perspective) 155, 210901 (2021) (invited) (selected as cover art)

# Optimal control theory in chemical reactions



S. Kawai, T. Komatsuzaki, **Bull. Chem. Soc. Japan 85**, 854 (2012); **J. Chem. Phys. 134**, 024317 (2011)



✓ Electronic structure theories (e.g., HOMO-LUMO theory, QM/MM)

✓ Energy landscape or reaction (conformation) network representation



2: Understand why and how fast reaction takes place (kinetic, dynamic) ✓ Transition state theories in Gas/Condensed Phase, or more in general in Phase Space

PNAS 98 7666 (2002); PRL 97, 028302 (2006); PRL 105, 048304, (2010); PRL 106, 054101 (2011); PRL 115, 093003 (2015); Nonlinearity 28, 2677 (2015); JCP (Perspective) 155, 210901 (2021)

#### 3: Constructing energy landscape/network from Experimental data (data science)

✓ Single Molecule Time series Analysis for Molecular Science

#### 4 : Accelerate designing experiments via adaptive feedback between measurement/computation and data analysis

(reinforcement learning) Mach. Learn. 109, 327-372 (2020); AISTATS, Volume 206, 29pages (2023). PNAS under revision



PNAS 104,19297 (2007); PNAS 105, 536 (2008); PRL 111, 58301(2013); Sci. Rep. 5, 9174 (2015); Nature Commun. 6,10223 (2015); JCP 148, 123325 (2018) (special issue on single molecule biophysics)



PNAS 103, 18551 (2006)

# Inferring cell-cell interactions from images

S. Sattari et al., **Sci. Adv. 8**(6), abj1720 (2022) (**Chosen as featured on-line article**)

U. Basak et al, PRE 102, 012404 (2020); JCP 154, 034901 (2021)



**Fluorescence imaging** 

# Facts and artifacts in interpreting dynamic disorder of enzymatic turnover reactions



Single molecule time series

C.-B. Li et al., **PRL 111**, 58301(2013)

#### Cell state defined by single cell Ramar Raman imaging

K. M. Helal, et al., **FEBS Lett. 593**, 2535-2544 (2019)(**Chosen as cover art & highlight article**) J. Nicholas Taylor, et al. *J. Phys. Chem.* B **123**, 4358-4372 (2019)





Let the Systems Speak for Themselves

Coordination of microtubules beyond single cells in sepals



S. Tsugawa et al., **AFM** (2016) (Chosen as highlighted article) L. Hong et al., **Dev. Cell 38**, 15 (2016) (Chosen as cover art & Highlight)

#### multidimensional Energy Landscape



A. Baba et al., **PNAS 104**,19297 (2007); **PCCP**, **13**, 1395 (2011); K. Kamagata et al., **JACS 134**, 11525 (2012); J.N. Taylor et al., **Sci. Rep. 5**, 9174 (2015) J. Alfermann et al., **Nature Chem. Bio. 13**, 1009 (2017)

# Nonequilibrium steady state network



Single molecule time series



C.-B. Li et al., **PNAS,105**, 536 (2008); C.-B. Li et al., **JPC B, 113**, 14732 (2009); T. Sultana et al. **JCP 139**, 245101(2013)

## **Energy Landscape: confe**

F. Rao and A. Caflisch, *J. Mol. Biol.* (2004) **342**,299 Krivov, Muff, Caflisch, Karplus , *JPCB* **112**, 8701 (2008



The complexity of

conformation space network

# Disconnectivity Graph: Visualization of Complexity of Energy Landscape



D. J. Wales, *Energy Landscapes* (Cambridge University Press, Cambridge, 2004). O. M. Becker and M. Karplus, J. Chem. Phys. **106**, 1495 (1997) R. Czerminski and R. Elber, J. Chem. Phys. **92**, 5580 (1990)

# 1. Background and Motivation

# 2. Transition Disconnectivity Graph

3.Network Transition States4.Lumping Theory to Preserve Hierarchical Timescales5.Outlooks

## Transition disconnectivity graph (TRDG)

S.V. Krivov & M. Karplus, Proc. Natl. Acad. Sci. USA **101** 14766 (2004) Krivov, Muff, Caflisch, Karplus , JPCB **112**, 8701 (2008)

Transforming a conformation space network into a new type of disconnectivity graph in terms of a network theory using rates among conformations







TRDG of  $\beta$ -hairpin at 360 K.

### **Transformation from Network to Energy Landscape**

S. V. Krivov and M. Karplus, J. Chem. Phys., 117, 10894 (2002); Proc. Natl. Acad. Sci. USA 101 14766 (2004)



Each sub region has its own bottleneck.

Energy Barrier  $\propto -k_B T \ln(k_{i \rightarrow j} P_i)$ 

Hierarchical grouping

free energy disconnectivity graph can be constructed by taking into account all possible pathways in the network

Free energy of the state *i* 
$$F_i = -k_B T \ln\left(\frac{\sum_j N_{i \to j}}{N}\right) = -k_B T \ln\left(\frac{N_i}{N}\right) = -k_B T \ln P_i$$

Kramers rate theory

$$k_{i \to j} \approx \frac{1}{\tau_0} \exp\left(-\frac{F_{ij} - F_i}{k_B T}\right)$$

 $\tau_0$ : the decay timescale of the autocorrelation for motion exerted by friction from the environment.

$$k_{i \to j} P_i = \left(\frac{N_{i \to j}}{N_i} \frac{1}{\tau_{obs}}\right) \left(\frac{N_i}{N}\right) = \frac{1}{\tau_0} \exp\left(-\frac{F_{ij} - F_i}{k_B T}\right) \exp\left(-\frac{F_i}{k_B T}\right) = \frac{1}{\tau_0} \exp\left(-\frac{F_{ij}}{k_B T}\right).$$
  
Timescale of observation  

$$F_{ij} = -k_B T \ln(\tau_0 k_{i \to j} P_i) = -k_B T \ln\left(\frac{N_{i \to j}}{N} \frac{\tau_0}{\tau_{obs}}\right)$$
in order to validate the free  
energy at the barrier,  $F_{ij} = F_{ji}$   
 $k_{i \to j} P_i = k_{j \to i} P_j$  should hold.

# Why does the minimum flux approach miss the "transition state" in network?



 $p[S] \gg p[S^C]$ 

When the space of optimization is the entire one, the minimum flux surface can be located near the "edge"

However, "Balanced min-cut is not unique r and can break balance in some cases"

KIIVOV-Karpius choice

**Balanced min-cut** 

# Why does the minimum flux approach miss the "transition state" in network?



When the space of optimization is the entire one, the minimum flux surface can be located near the "edge" Can one infer the region of dividing surface over an entire network?

\*Krivov-Karplus' choice Balanced min-cut Background and Motivation
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# TS in network is defined as a dividing surface across which passage is slowest







$$\min_{S} \left[ \max\left\{ \frac{p_{\Delta t}[S^{c}, S]}{p[S]}, \frac{p_{\Delta t}[S, S^{c}]}{p[S^{c}]} \right\} \right]$$

$$p[S]\left(=\sum_{s_i\in S}p_{eq}[s_i]\right)$$





By Maeda & Taketsugu, GRRM/SC-AFIR with CCSD(T)/jun-cc-pVTZ//Mo62X/6-311+G(2d,p)

## Kinetic disconnectivity graph and Markov kinetics





Experimental data F. W. Schuler and G. W. Murphy, J. Am. Chem. Soc. 72, 3155 (1950).



Background and Motivation
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dividing surface in phase space reactant product Phase Space Geometry, of Chemical Reactions Nagahata et al., J. Chem. Phys. (Perspective) 155, 210901 (2021)

Dynamics



Representation of Kinetic Hierarchies not necessarily based on equilibrium

Free from stationarity, timescale separation, Predefining subnetworks



D. J. Wales, Curr. Opin. Struct. Biol. **20**, 3 (2010).



# Lumpability diagram vs kinetic DG

### Lumpability diagram

Identifying  $\epsilon$ -indistinguishability of dynamics under a given timescale of observation.

### **Computational costs:**

Solving ODE: diagonalization  $\mathcal{O}(n^3)$ Find  $\Delta t^{\epsilon}$ : matrix multiplication  $\mathcal{O}(n^3)$ matrix arithmetic  $\mathcal{O}(n^2)$ multi-precision root finding  $\times \mathcal{O}(n^2)$ Constructing diagram complete linkage  $\mathcal{O}(n^2)$ 

### Kinetic disconnectivity graph

Showing whether each subnetwork is connected under a given threshold of timescale

### **Computational costs:**

Computing rate constants: arithmetic  $\times O(n^2)$ Constructing diagram single linkage  $O(n^2)$ 



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