Information Geometry for Ladders and Steppingstones

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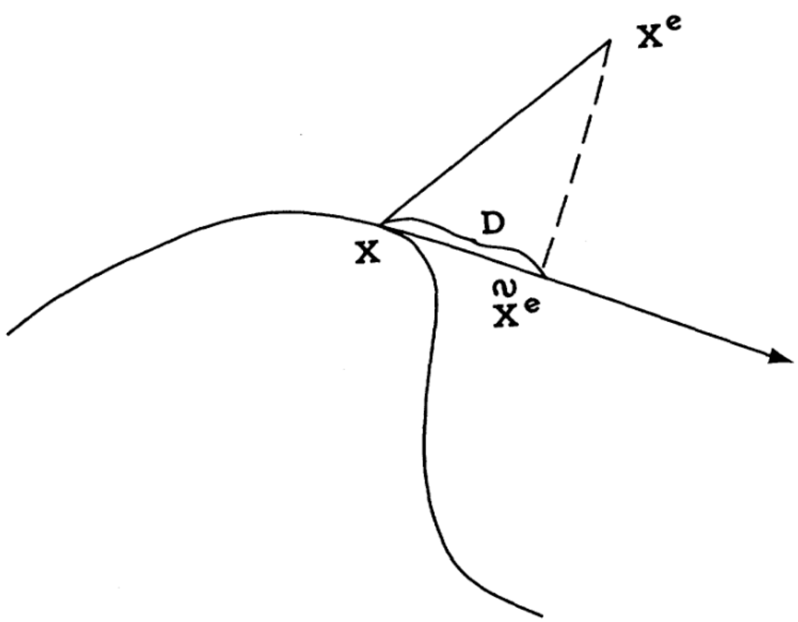
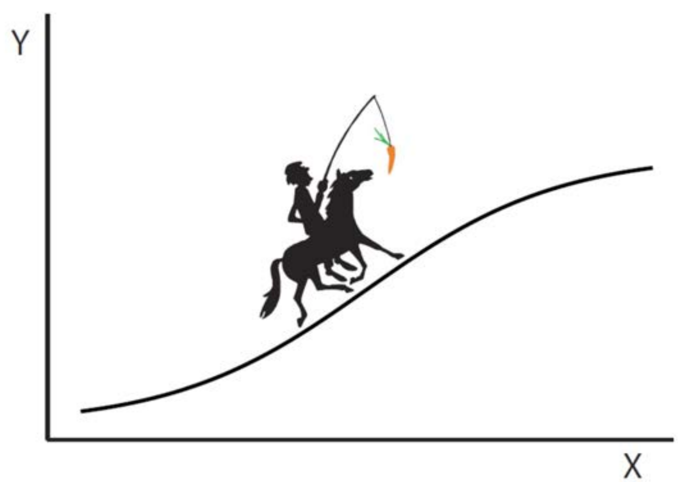
Real time counterparts of quasistatic processes may be analyzed as a sequence of relaxations.



Given a path, set stages along path.
Then iterate: Equilibrate to the next stage.

For small stages, dissipated work in move
= (distance moved)^2/2

Horse-Carrot Theorem (1983)

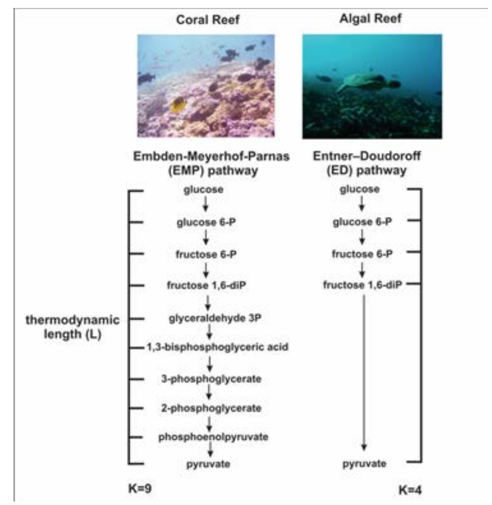


$$\text{Min Dissipation} = \frac{L^2}{2K}$$

L = thermodynamic length of path
K = number of equilibrations

The Ladder Theorem (2017)

Adding a new relaxation step by braking one relaxation into two relaxations, always decreases the dissipation, provided only that the inserted state be somewhere in-between the start and end points of the relaxation being divided.

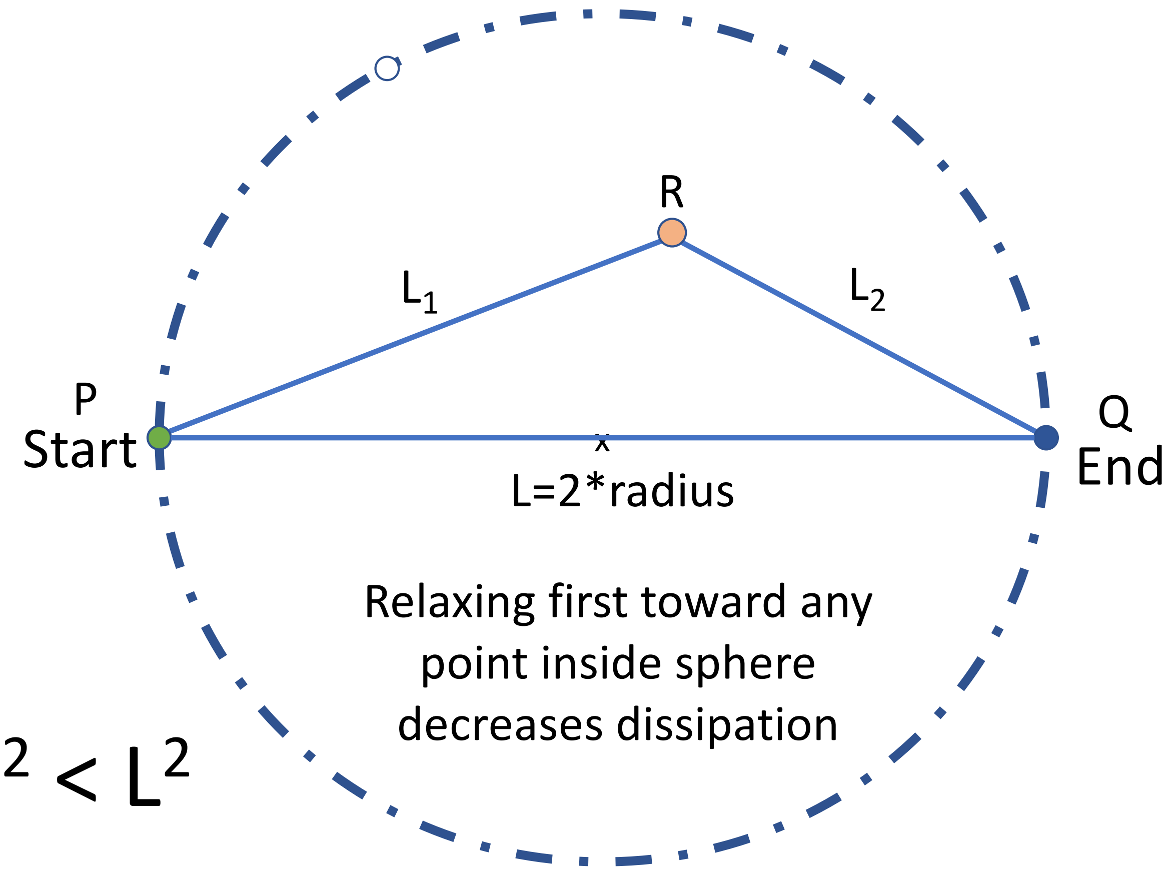


Steppingstone states

We say that a state R is *in-between* two states P and Q provided the two-step relaxation $P \to R$ followed by $R \to Q$ produces less entropy than the one step relaxation $P \to Q$.

$$\text{Gain} = \Delta S^U_{PQ} - \Delta S^U_{PR} - \Delta S^U_{RQ}$$

Recall that for small relaxations, dissipation is half the distance squared.


R
L1
L2
P
Start
Q
End
x
L=2*radius
Relaxing first toward any
point inside sphere
decreases dissipation

$$L_1^2 + L_2^2 < L^2$$

Realizations at Many Levels

- Metabolic pathways
- Macro Thermodynamic states
- Statistical Thermodynamics
- Quantum Mechanics
- Exponential Family

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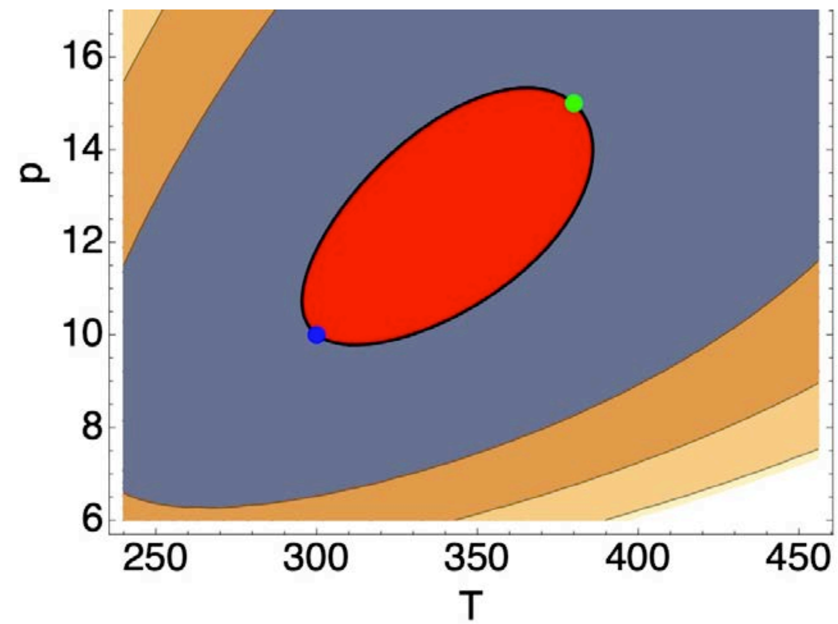

Red region = steppingstone states for a relaxation in an ideal gas

Karl Heinz Hoffmann visiting San Diego this spring gave a beautiful one hour talk two days after hearing of the topic.

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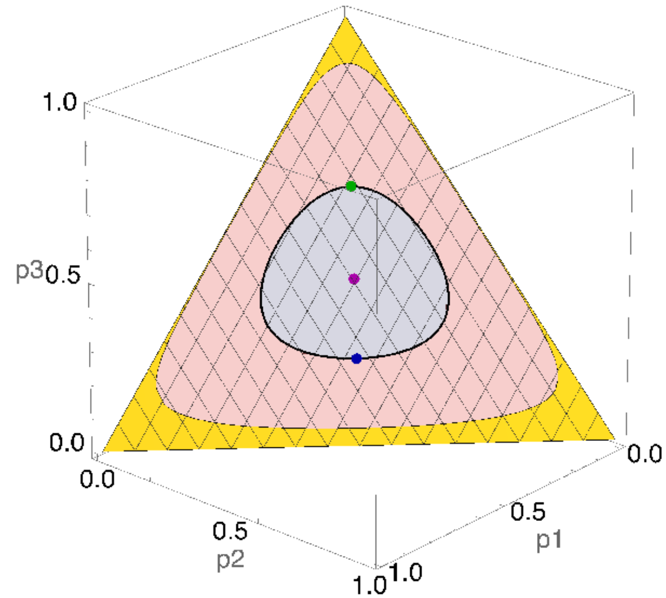

Grey region = steppingstone states for a relaxation in a three level system

Also KHH graphics

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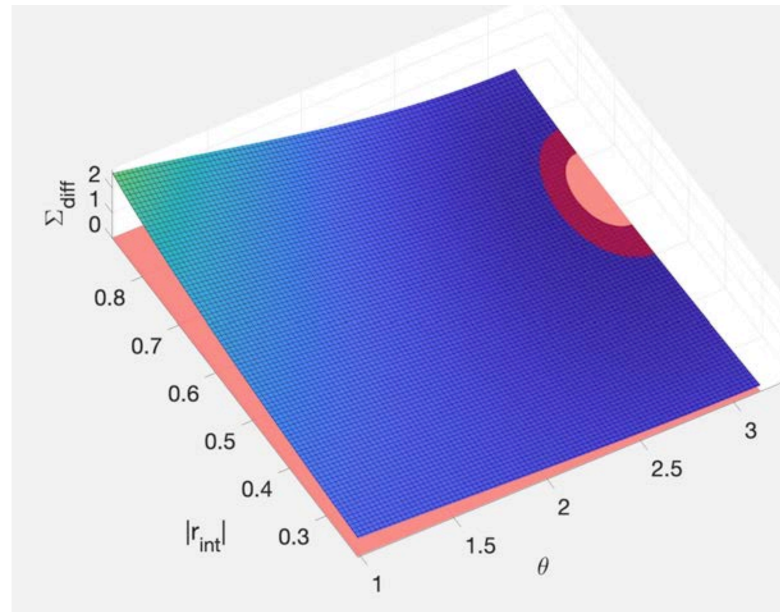

Pink region = steppingstone states for relaxation in a qubit

Graphics by Roie Dann

Realized at many levels due to the geometry

- Hessian Geometry
  - Semi-Riemannian metric from the second derivative of a function
  - Dual flat coordinates connected by Legendre transforms
    - extensive ←→ intensive
    - p ←→ ln(p)

Gain = $KL(p,q) - KL(p,r) - KL(r,q) = \sum_i (p_i - r_i)\log(r_i/q_i)$

$$(V_1 - V_2) * (p_2 - p_3) + (N_1 - N_2) * (\mu_2 - \mu_3) + (S_1 - S_2) * (T_2 - T_3)$$

$$(V_1, N_1, S_1) \to (V_3, N_3, S_3)$$

Take-home lesson:

- Adding substeps to a relaxation reduces its dissipation.
- A substep can use any in-between state.
- This defined in-between states, i.e., steppingstones.